Programa de Pós-Graduação em Engenharia Civil

Universidade Federal de Alagoas

MODELOS CONSTITUTIVOS

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TERMOELASTICIDADE

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Corpo elástico linear, homogêneo e isótropo

A variação de temperatura não afeta as constantes do material

Sólido sem vínculos

- Tensão nula
- Não ocorre mudança de forma
- Somente acontece variações dimensionais

$$\Delta L = L\alpha\Delta T$$

$$\varepsilon_T = \frac{\Delta L}{L} = \alpha\Delta T$$

$\alpha =$ coeficiente de dilatação térmica linear do material

Em geral, para um corpo tridimensional com vínculos, tem-se:

$$\boxed{\varepsilon_{ij} = \varepsilon_{ij}^{\sigma} + \varepsilon_{ij}^{T}}$$

$$\boxed{\varepsilon_{11} = \frac{1}{E}\left[\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})\right] + \alpha\Delta T}$$

$$\boxed{\gamma_{12} = 2\varepsilon_{12} = \frac{1}{G}\sigma_{12}}$$

$$\boxed{\varepsilon_{22} = \frac{1}{E}\left[\sigma_{22} - \nu(\sigma_{11} + \sigma_{33})\right] + \alpha\Delta T}$$

$$\boxed{\gamma_{13} = 2\varepsilon_{13} = \frac{1}{G}\sigma_{13}}$$

$$\boxed{\varepsilon_{33} = \frac{1}{E}\left[\sigma_{33} - \nu(\sigma_{11} + \sigma_{22})\right] + \alpha\Delta T}$$

$$\boxed{\gamma_{23} = 2\varepsilon_{23} = \frac{1}{G}\sigma_{23}}$$

$$
\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} 1/E & \nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} + \begin{bmatrix} \alpha\Delta T \\ \alpha\Delta T \\ \alpha\Delta T \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} + \alpha\Delta T\delta_{ij}
$$

Relações de Duhamel-Newmann

$$
\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} - \beta\Delta T\delta_{ij}
$$

$$
\beta = \alpha(3\lambda + 2G)
$$

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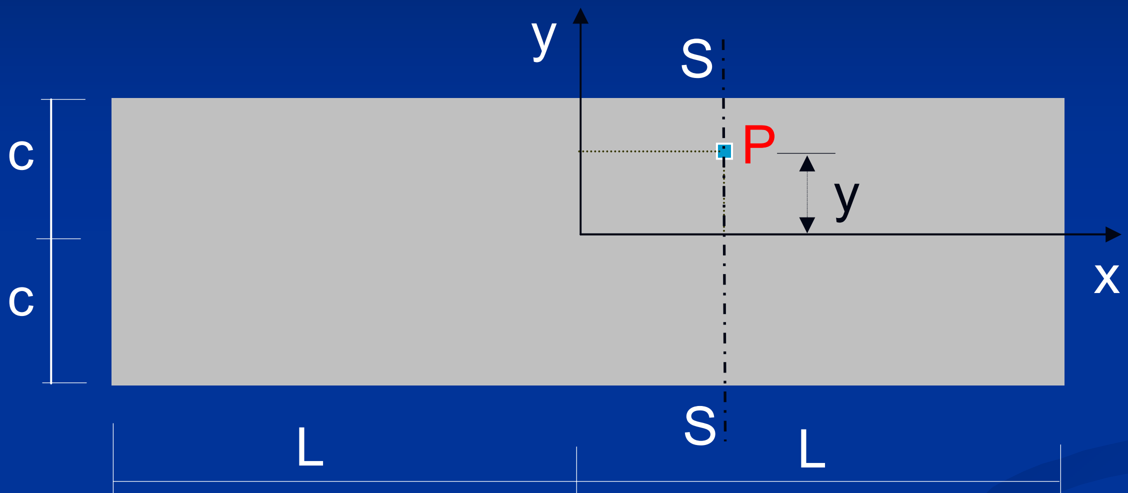
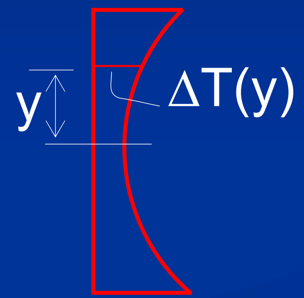
CHAPA SOB UMA VARIAÇÃO DE TEMPERATURA SIMÉTRICA

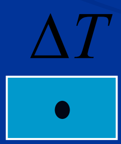




Variação de temperatura

$$\Delta T(y) = \Delta T(-y)$$

No ponto P:

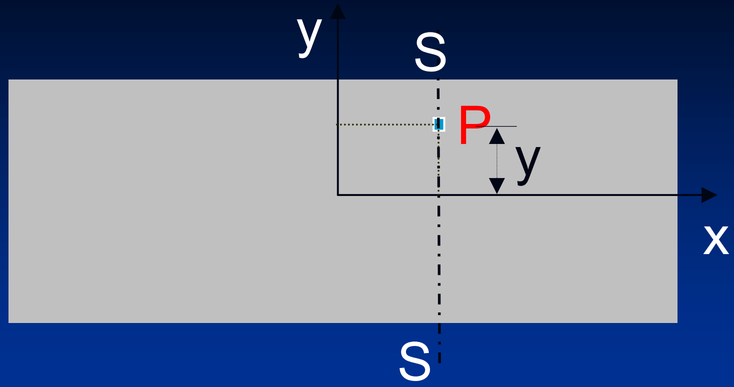


+

$$\boxed{\varepsilon(y) = \varepsilon_T(y) + \varepsilon_e(y)}$$

$$\varepsilon_T(y) = \alpha \Delta T(y)$$

$$\varepsilon_e(y) = \frac{\sigma(y)}{E}$$

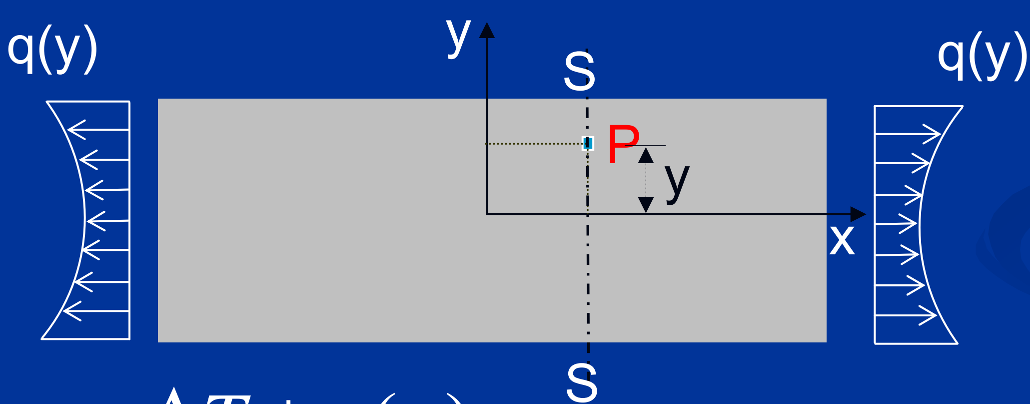


Em geral,

$$\varepsilon(y) = \alpha \Delta T(y) + \frac{\sigma(y)}{E}$$

$$\sigma(y) = E\left[\varepsilon(y) - \alpha \Delta T(y)\right]$$

Adicionar uma carga externa q(y) de modo que resulte



$$\varepsilon'(y) = 0$$

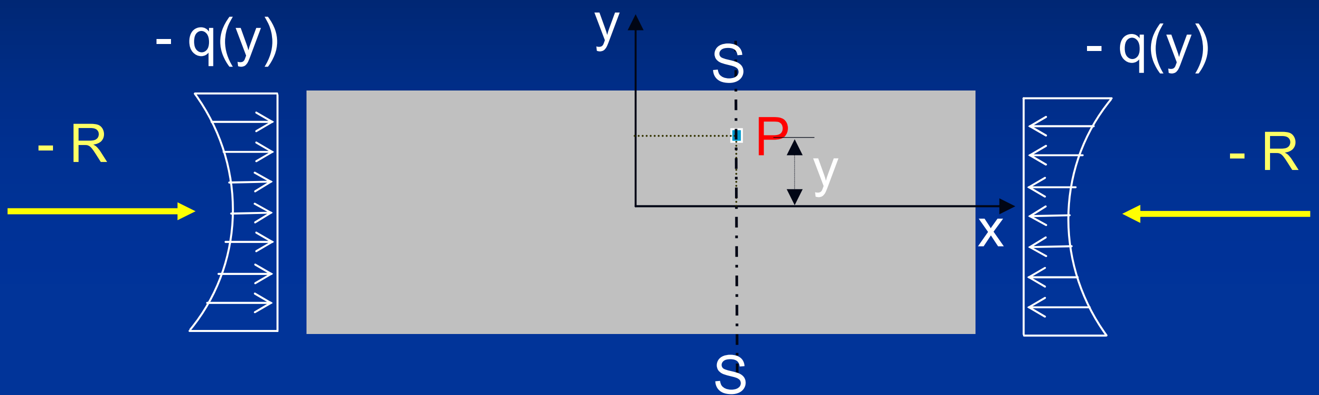
$$\sigma'(y) = -E\alpha \Delta T(y)$$

$$\Delta T + q(y)$$

Força resultante na seção S-S:

$$R = -\int_{-c}^{c} E\alpha \Delta T(y)\, dy$$

Aplicando uma carga: – q(y)



$$\sigma'' = -\frac{R}{2c} = \frac{1}{2c}\int_{-c}^{c} E\alpha\Delta T(y)dy$$

L grande

$$\varepsilon'' = \varepsilon$$

Por superposição de efeitos, a tensão no ponto P:

$$\sigma = \sigma' + \sigma''$$

$$\sigma(y) = -E\alpha\Delta T(y) + \frac{1}{2c}\int_{-c}^{c} E\alpha\Delta T(y)\,dy$$

A deformação no ponto P da chapa:

$$\varepsilon(y) = \alpha\Delta T(y) + \frac{\sigma(y)}{E}$$

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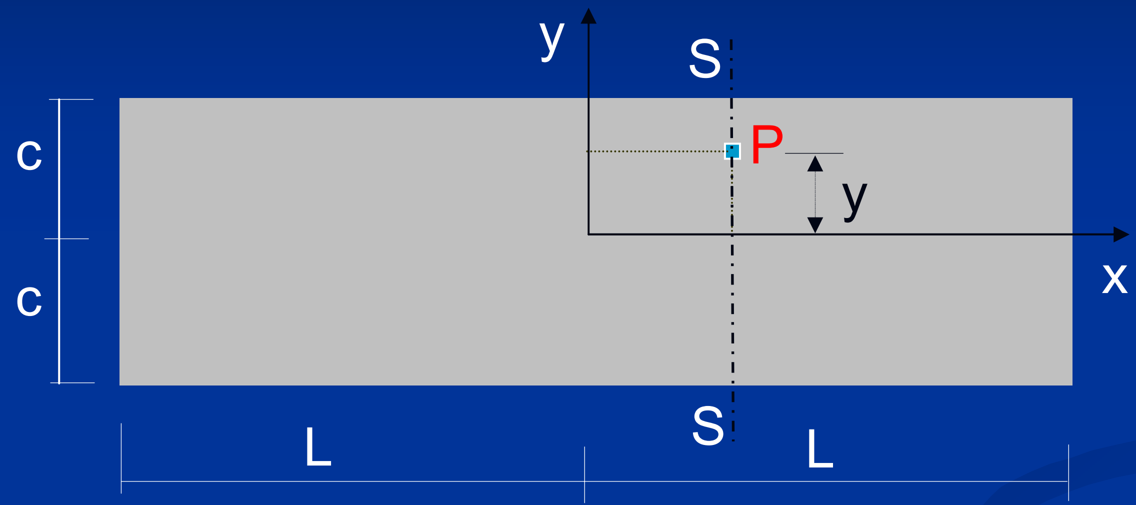
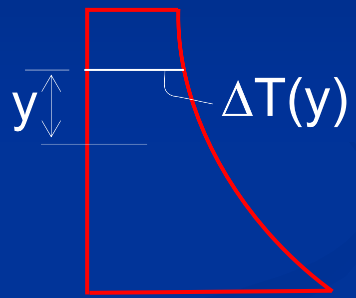
CHAPA SOB UMA VARIAÇÃO DE TEMPERATURA NÃO SIMÉTRICA





Variação de temperatura

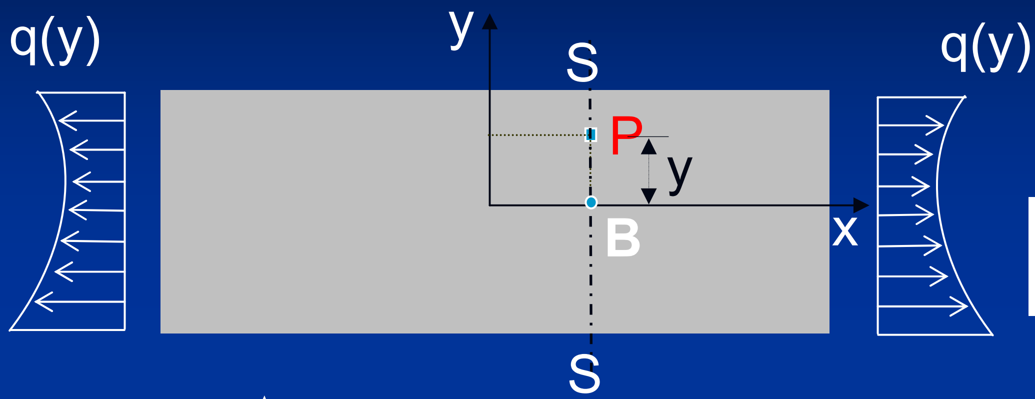
No ponto P:

$$\boxed{\varepsilon(y) = \varepsilon_T(y) + \varepsilon_e(y)}$$

$$\varepsilon_T(y) = \alpha \Delta T(y)$$

$$\varepsilon_e(y) = \frac{\sigma(y)}{E}$$

Adicionar uma carga externa q(y) de modo que resulte



$$\Delta T + q(y)$$

$$\varepsilon'(y) = 0$$

$$\sigma'(y) = -E\alpha\Delta T(y)$$

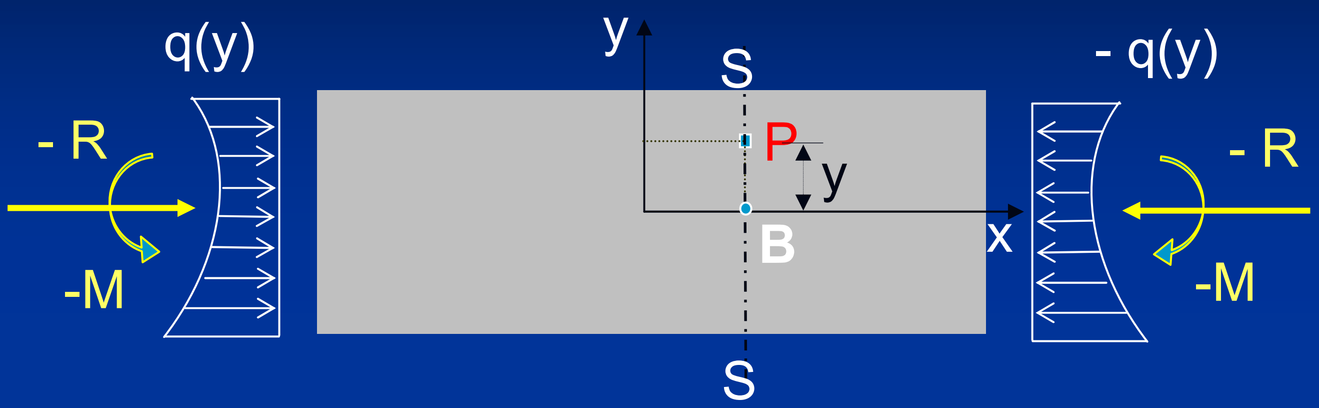
Força resultante na seção S-S:

$$R = -\int_{-c}^{c} E\alpha\Delta T(y)dy$$

Momento em relação ao ponto B:

$$M = -\int_{-c}^{c} E\alpha\Delta T(y)ydy$$

Aplicando uma carga: – q(y)



$$\sigma'' = \frac{1}{2c}\int_{-c}^{c} E\alpha\Delta T(y)dy + \frac{y}{\frac{(2c)^3}{12}}\int_{-c}^{c} E\alpha\Delta T(y)ydy$$

$$\varepsilon'' = \varepsilon$$

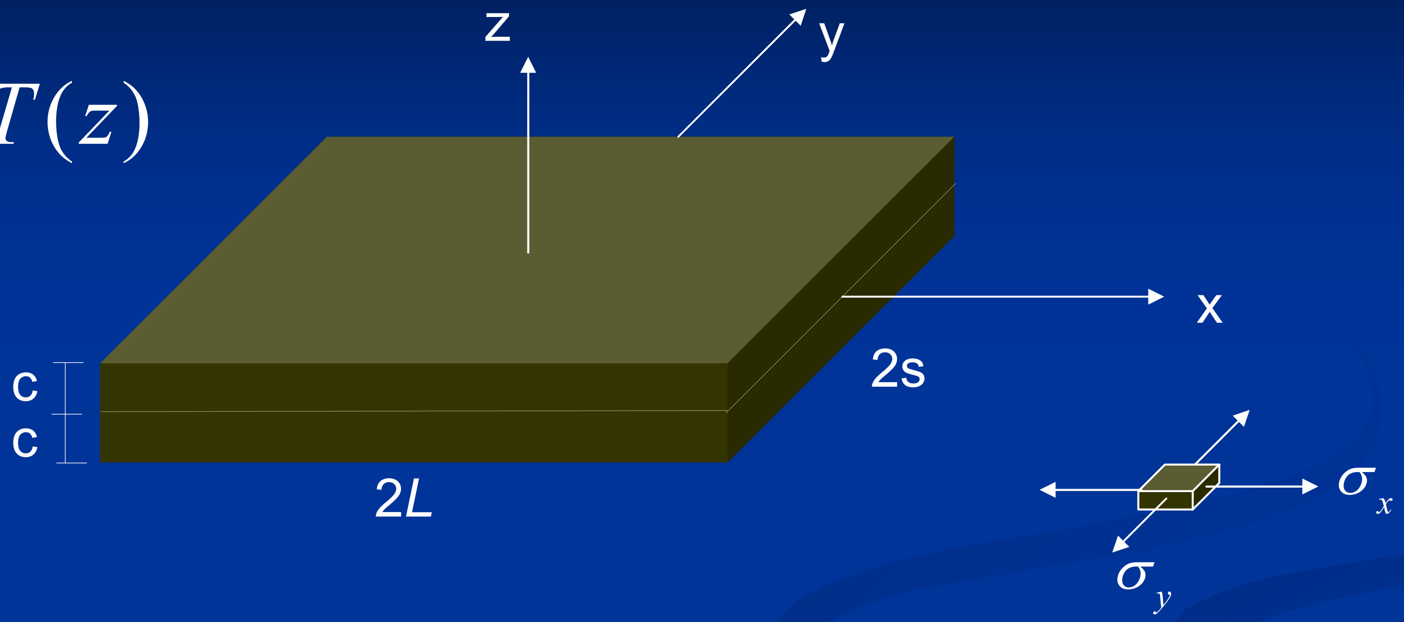
Por superposição de efeitos, a tensão no ponto P:

$$\sigma = \sigma' + \sigma''$$

$$\sigma(y) = -E\alpha\Delta T(y) + \frac{1}{2c}\int_{-c}^{c} E\alpha\Delta T(y)dy + \frac{3y}{2c^3}\int_{-c}^{c} E\alpha\Delta T(y)ydy$$

PLACA SUBMETIDA À VARIAÇÃO DE TEMPERATURA

$$\Delta T(z)$$



$$\varepsilon_x = \alpha \Delta T(z) + \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$

$$\varepsilon_y = \alpha \Delta T(z) + \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}$$

$$\varepsilon_x = \varepsilon_y = 0 \Rightarrow \frac{1}{E}\left(\sigma_x - \sigma_y\right) - \frac{\nu}{E}\left(\sigma_y - \sigma_x\right) = 0 \Rightarrow$$

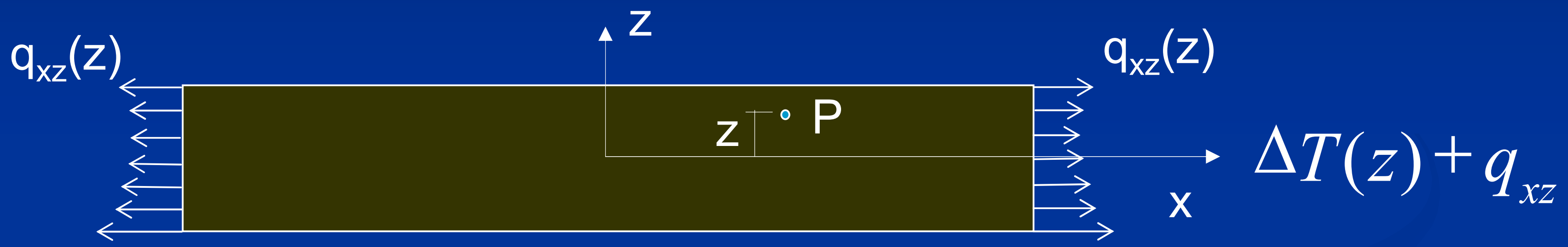
$$\sigma_x = \sigma_y \Rightarrow \sigma_x\left(\frac{1-\nu}{E}\right) = -\alpha \Delta T(z)$$

Então, para $\varepsilon_x = \varepsilon_y = 0:$

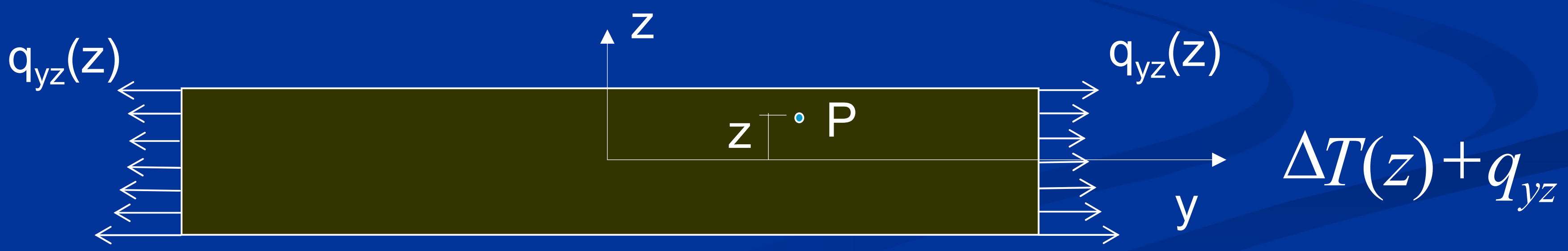
$$\boxed{\sigma_x = \sigma_y = -\frac{E\alpha\Delta T(z)}{1-\nu}}$$

$$\boxed{\varepsilon_x = \varepsilon_y = 0}$$

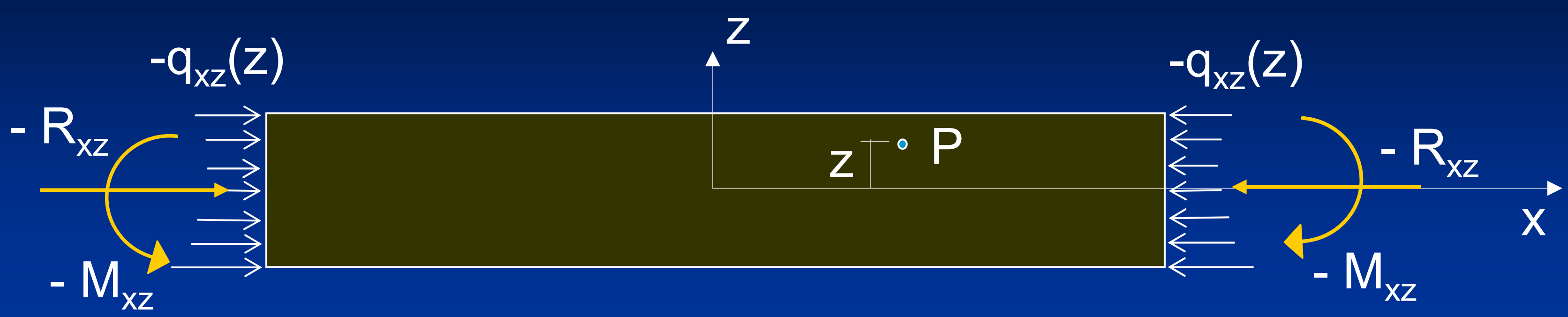
PLANO xz



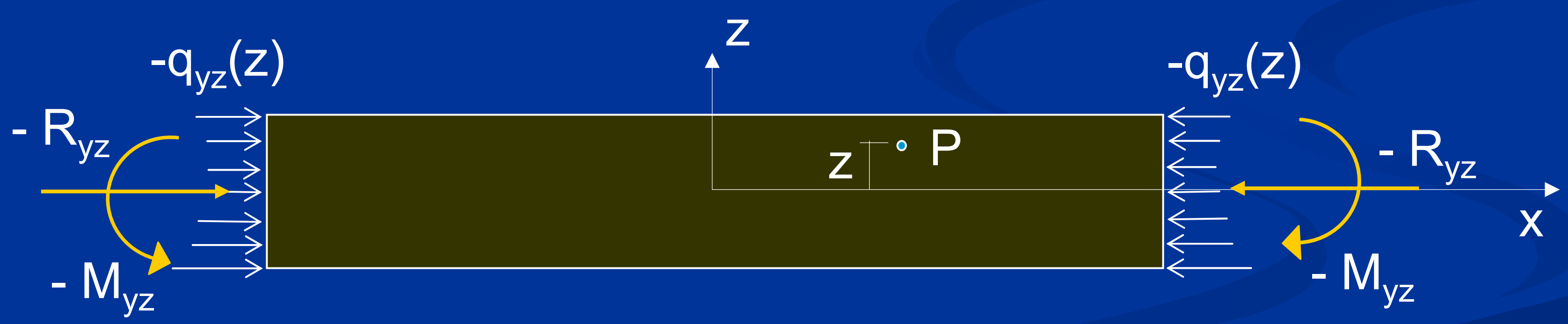
PLANO yz



PLANO xz



PLANO yz



Sendo

$$R_{xz} = -\int_{-c}^{c} \frac{E\alpha\Delta T(z)}{1-\nu} dz \qquad M_{xz} = -\int_{-c}^{c} \frac{E\alpha\Delta T(z)}{1-\nu} z dz$$

$$R_{yz} = -\int_{-c}^{c} \frac{E\alpha\Delta T(z)}{1-\nu} dz \qquad M_{yz} = -\int_{-c}^{c} \frac{E\alpha\Delta T(z)}{1-\nu} z dz$$

$$\boxed{\sigma_x = -\frac{E\alpha\Delta T(z)}{1-\nu} + \frac{1}{2c}\int_{-c}^{c} \frac{E\alpha\Delta T(z)}{1-\nu} dz + \frac{3z}{2c^3}\int_{-c}^{c} \frac{E\alpha\Delta T(z)}{1-\nu} z dz}$$

$$\boxed{\sigma_y = \sigma_x}$$